Paper Reference(s)

6678

Edexcel GCE

Mechanics M2

Advanced Level

Friday 29 January 2010 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Pink or Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of g is required, take $g = 9.8$ m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 8 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

1. A particle P moves along the x-axis. At time t seconds the velocity of P is v m s^{-1} in the positive x-direction, where $v = 3t^2 - 4t + 3$. When $t = 0$, P is at the origin O. Find the distance of P from O when P is moving with minimum velocity.

(8)

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2. Two particles, P, of mass $2m$, and Q, of mass m, are moving along the same straight line on a smooth horizontal plane. They are moving in opposite directions towards each other and collide. Immediately before the collision the speed of P is $2u$ and the speed of Q is u. The coefficient of restitution between the particles is e, where $e < 1$. Find, in terms of u and e,

(i) the speed of P immediately after the collision,

(ii) the speed of Q immediately after the collision.

(7)

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3. A particle of mass 0.5 kg is projected vertically upwards from ground level with a speed of 20 m s^{-1}. It comes to instantaneous rest at a height of 10 m above the ground. As the particle moves it is subject to air resistance of constant magnitude R newtons. Using the work-energy principle, or otherwise, find the value of R.

(6)

4.

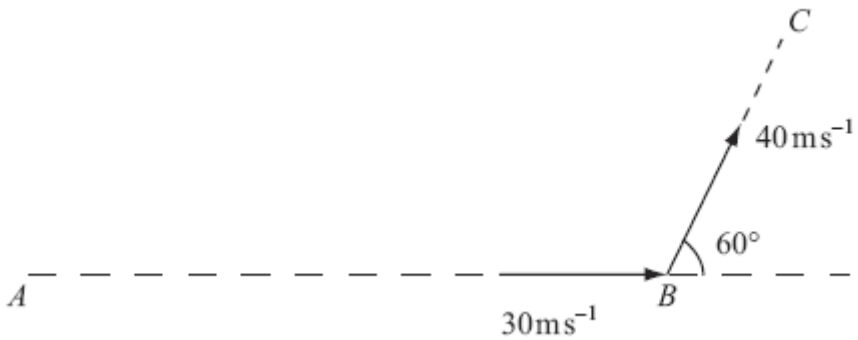

Figure 1

The points A, B and C lie in a horizontal plane. A batsman strikes a ball of mass 0.25 kg. Immediately before being struck, the ball is moving along the horizontal line AB with speed 30 m s^{-1}. Immediately after being struck, the ball moves along the horizontal line BC with speed 40 m s^{-1}. The line BC makes an angle of 60° with the original direction of motion AB, as shown in Figure 1.

Find, to 3 significant figures,

(i) the magnitude of the impulse given to the ball,

(ii) the size of the angle that the direction of this impulse makes with the original direction of motion AB.

(8)

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5. A cyclist and her bicycle have a total mass of 70 kg. She cycles along a straight horizontal road with constant speed 3.5 m s^{-1}. She is working at a constant rate of 490 W.

(*a*) Find the magnitude of the resistance to motion.

(4)

The cyclist now cycles down a straight road which is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{1}{14}$, at a constant speed U m s^{-1}. The magnitude of the non-gravitational resistance to motion is modelled as $40U$ newtons. She is now working at a constant rate of 24 W.

(*b*) Find the value of U.

(7)

6.

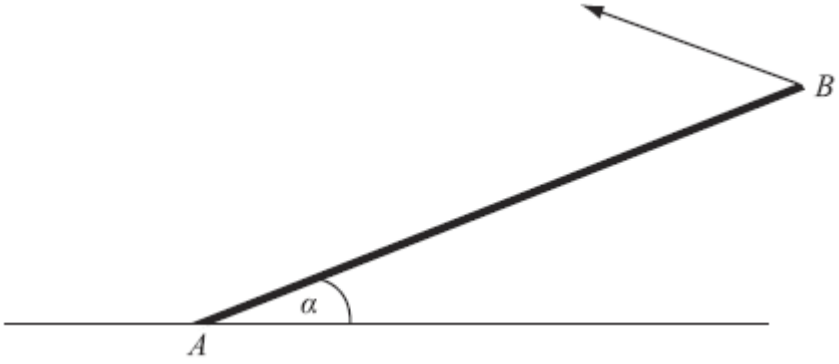

Figure 2

A uniform rod AB, of mass 20 kg and length 4 m, rests with one end A on rough horizontal ground. The rod is held in limiting equilibrium at an angle α to the horizontal, where $\tan\alpha = \frac{3}{4}$, by a force acting at B, as shown in Figure 2. The line of action of this force lies in the vertical plane which contains the rod. The coefficient of friction between the ground and the rod is 0.5.

Find the magnitude of the normal reaction of the ground on the rod at A.

(7)

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7. [*The centre of mass of a semi-circular lamina of radius r is* $\frac{4r}{3\pi}$ *from the centre.*]

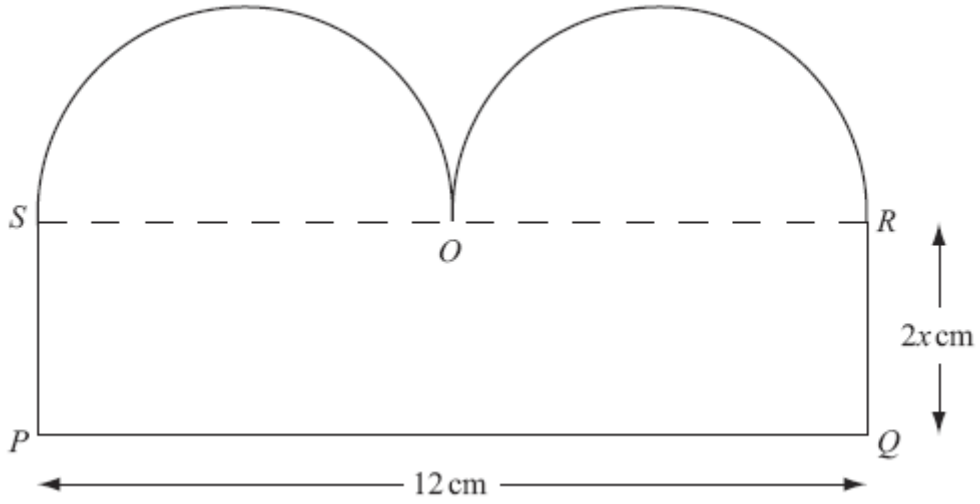


Figure 3

A template *T* consists of a uniform plane lamina *PQROS*, as shown in Figure 3. The lamina is bounded by two semicircles, with diameters *SO* and *OR*, and by the sides *SP*, *PQ* and *QR* of the rectangle *PQRS*. The point *O* is the mid-point of *SR*, *PQ* = 12 cm and *QR* = 2*x* cm.

(*a*) Show that the centre of mass of *T* is a distance $\frac{4\left|2x^2-3\right|}{8x+3\pi}$ cm from *SR*.

(7)

The template *T* is freely suspended from the point *P* and hangs in equilibrium.

Given that $x = 2$ and that θ is the angle that *PQ* makes with the horizontal,

(*b*) show that $\tan\theta = \frac{48+9\pi}{22+6\pi}$.

(4)

8. [*In this question* **i** *and* **j** *are unit vectors in a horizontal and upward vertical direction respectively*.]

A particle P is projected from a fixed point O on horizontal ground with velocity $u(\mathbf{i} + c\mathbf{j})$ m s^{-1}, where c and u are positive constants. The particle moves freely under gravity until it strikes the ground at A, where it immediately comes to rest. Relative to O, the position vector of a point on the path of P is $(x\mathbf{i} + y\mathbf{j})$ m.

(a) Show that

$$y = cx - \frac{4.9x^2}{u^2}.$$

(5)

Given that $u = 7$, $OA = R$ m and the maximum vertical height of P above the ground is H m,

(b) using the result in part (a), or otherwise, find, in terms of c,

(i) R

(ii) H.

(6)

Given also that when P is at the point Q, the velocity of P is at right angles to its initial velocity,

(c) find, in terms of c, the value of x at Q.

(6)

TOTAL FOR PAPER: 75 MARKS

END

edexcel

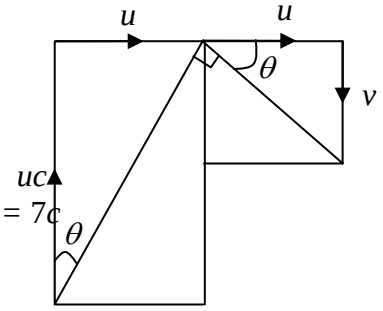
January 2010
6678 Mechanics M2
Mark Scheme

Question Number	Scheme	Marks
Q1.	$\frac{\mathrm{d}v}{\mathrm{d}t} = 6t - 4$	M1 A1
	$6t - 4 = 0 \Rightarrow t = \frac{2}{3}$	M1 A1
	$s = \int 3t^2 - 4t + 3 \,\mathrm{d}t = t^3 - 2t^2 + 3t \ (+c)$	M1 A1
	$t = \frac{2}{3} \Rightarrow s = -\frac{16}{27} + 2$ so distance is $\frac{38}{27}$ m	M1 A1
		[8]
Q2.	$2u \rightarrow$ $\quad$ $\leftarrow u$ $2m \rightarrow$ $\quad$ $m \rightarrow$ v_1 $\quad$ v_2	
	CLM: $4mu - mu = 2mv_1 + mv_2$	M1 A1
	i.e. $3u = 2v_1 + v_2$	
	NIL: $3eu = -v_1 + v_2$	M1 A1
	$v_1 = u(1 - e)$	DM1 A1
	$v_2 = u(1 + 2e)$	A1
		[7]
Q3.	$\frac{1}{2}$ x 0.5 x 20^2 ; $0.5g$ x 10	B1 B1
	$10R = \frac{1}{2}$ x 0.5 x $20^2 - 0.5g$ x 10	M1 A1
	$\Rightarrow R = 5.1$	DM1 A1
		[6]

Question Number	Scheme	Marks
Q4.	(i) $I\uparrow = 0.25\times 40\sin 60 = 5\sqrt{3}$ (8.66) one component $I\leftarrow = 0.25(-20+30) = 2.5$ both $\lvert I\rvert = \sqrt{75+6.25} = 9.01$ (Ns)	M1 A1 M1 A1 (4)
	(ii) $\dfrac{\sin\theta}{40} = \dfrac{\sin 60^o}{\sqrt{1300}}$ $\theta = 106^o$ (3 s.f.)	M1 A1
	or $\tan\theta = \pm\dfrac{5\sqrt{3}}{2.5}$ oee $\theta = 106°$	M1 A1 (4)
		[8]
	Alternative to 4(i) Use of $\mathbf{I} = m(\mathbf{v}-\mathbf{u})$	M1
	$30^2 + 40^2 - 2 \text{ x } 30 \text{ x } 40\cos 60^o$ (= 1300)	M1 A1
	$I = 0.25\sqrt{1300} = 9.01$ N s (3 s.f.)	A1
	2nd Alternative to 4(i) $\mathbf{u} = 30\mathbf{i}$, $\mathbf{v} = 40\cos 60\mathbf{i} + 40\sin 60\mathbf{j} = 20\mathbf{i} + 20\sqrt{3}\mathbf{j}$ $\mathrm{I} = \dfrac{1}{4}(-10\mathbf{i} + 20\sqrt{3}\mathbf{j}) = -2.5\mathbf{i} + 5\sqrt{3}\mathbf{j}$	 M1 A1 etc

Question Number	Scheme	Marks
Q5.	(a) $\frac{490}{3.5} - R = 0$ $R = 140$ N	B1 M1 A1 A1 (4)
	(b) $\frac{24}{u} + 70g.\frac{1}{14} - 40u = 0$ $40u^2 - 49u - 24 = 0$ $(5u - 8)(8u + 3) = 0$ $u = 1.6$	B1 M1 A2,1,0 DM1 DM1 A1 (7) **[11]**
Q6.	$m(B): R \times 4\cos\alpha = F \times 4\sin\alpha + 20g \times 2\cos\alpha$ Use of $F = \frac{1}{2}R$ Use of correct trig ratios R = 160N or 157N	M1 A2 M1 B1 DM1 A1 **[7]**

Question Number	Scheme	Marks
Q7.	(a)	
	Rectangle: $24x$; Semicircles: 4.5π; 4.5π; Template, T: $24x + 9\pi$	B2
	x; $\frac{4 \text{ x } 3}{3\pi}$; $\frac{4 \text{ x } 3}{3\pi}$; $\bar{x}$	B2
	$24x^2 - 4.5\pi \text{ x } \left(\frac{4 \text{ x } 3}{3\pi}\right) - 4.5\pi \text{ x } \left(\frac{4 \text{ x } 3}{3\pi}\right) = (24x + 9\pi)\bar{x}$	M1 A1
	distance $= \left\|\bar{x}\right\| = \frac{4\left\|2x^2 - 3\right\|}{(8x + 3\pi)}$ **	A1 (7)
	(b)	
	When $x = 2$, $\left\|\bar{x}\right\| = \frac{20}{16 + 3\pi}$	B1
	$\tan\theta = \frac{6}{4 - \left\|\bar{x}\right\|} = \frac{6}{4 - \frac{20}{16 + 3\pi}}$	M1 A1
	$= \frac{48 + 9\pi}{22 + 6\pi}$.	A1 (4)
		[11]

Question Number	Scheme	Marks
Q8.	(a) $x = ut$	B1
	$y = cut - 4.9t^2$	M1 A1
	eliminating t and simplifying to give $y = cx - \frac{4.9x^2}{u^2}$ **	DM1 A1 (5)
	(b)(i) $0 = cx - \frac{4.9x^2}{u^2}$	M1
	$0 = x(c - \frac{4.9x}{u^2}) \Rightarrow R = \frac{u^2 c}{4.9} = 10c$	M1 A1
	(ii) When x = 5c, $y = H$	M1
	$= 5c^2 - \frac{(5c)^2}{10} = 2.5c^2$	M1 A1 (6)
	(c) $\frac{dy}{dx} = c - \frac{9.8x}{u^2} = c - \frac{x}{5}$	M1 A1
	When x = 0, $\frac{dy}{dx} = c$	B1
	So, $c - \frac{x}{5} = \frac{-1}{c}$	DM1 A1
	$x = 5(c + \frac{1}{c})$	A1 (6)
		[17]
	Alternative to 8(c)  $\tan\theta = \frac{u}{cu} = \frac{1}{c} = \frac{v}{u}$	B1
	$\Rightarrow v = \frac{u}{c} = \frac{7}{c}$	M1 A1
	$v = u + at$; $-\frac{7}{c} = 7c - 9.8t$	M1
	$t = \frac{7}{9.8}(c + \frac{1}{c})$	A1
	$x = ut = 7t$; $x = 5(c + \frac{1}{c})$	A1